IJBES

International Journal of Built Environment and Sustainability
Published by Penerbit UTM Press, Universiti Teknologi Malaysia
IJBES 8(1)/2021, 29-45

Spreadsheet Iteration of Reversionary Leasehold Rental Growth Rate Within The Framework of Explicit DCF Appraisals

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ABSTRACT

Income growth rates are required to justify decisions and strategies for property investments. Although existing studies addressed this phenomenon in freehold investments, a relative question regarding the determination of rental growth rates of leasehold investment properties valued part-way through rent review periods has not been addressed before now. This study examined the spreadsheet-assisted scenario analysis tools and techniques that are required for the determination of rental growth rates of leasehold investment properties valued part-way through rent review periods. A precursor to the scenario analysis was the development of a hybrid leasehold DCF valuation model arising from the equation of the formula for reversionary leasehold equivalent yield valuation to the formula for reversionary leasehold growth explicit DCF valuation model; thereby culminating into the identification of four unknown variables comprising the all risks yield and the implied growth rates of leasehold cash inflows and cash outflows which were subsequently derived using the solver tool of Excel®. From a total of eleven scenarios generated, the 9th successive scenario produced optimal results indicating zero slack between iterated and calculated values for the growth rates of leasehold cash inflows and cash outflows respectively. With recourse to the hybrid leasehold DCF valuation model, the spreadsheet-assisted scenario was found to produce mathematically valid growth rates that justify the valuation of leasehold investment properties part-way through rent review periods. The value of this research is the analytical tools and rigour it avails investors seeking income returns and growth from reversionary leasehold property as an instance of terminable investments.

Article History

Received: *16 April 2020*
Received in revised form: *19 August 2020*
Accepted: *07 December 2020*
Published Online: *31 December 2020*

Keywords:

Explicit DCF appraisal, Equivalent yield valuation, Implied rental growth, Reversionary leaseholds, Scenario analysis

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DOI: 10.11113/ijbes. v8.n1.576

1. Introduction

Notwithstanding that the limited time horizon of leasehold investments may not be attractive compared to freeholds, there are investors who still make leasehold investment decisions on the basis of specific factors such as short-term profits, outperformance of a competing investment, asset-liability matching, stability and security of returns, and expectations of future rental growth (Sayce *et al.*, 2006). Furthermore, it is possible that some investors might be attracted to leasehold investments as a result of personal or corporate considerations and macroeconomic factors beyond their control. Notwithstanding that leaseholds are terminable investments and ideally prone to capital value depreciation over time (Fraser, 1993), it is not impossible to rule out the chance of cyclical movements in the economy that might warrant capital value appreciation in the leasehold market in favour of speculators and arbitrageurs. In most cases however, there might be an appeal for cash flow growth potential of a leasehold investment to warrant "purchase-" or "hold" decision throughout the unexpired term of the grant.

When expected rental growth rate is implied in a transaction, it is appropriately called the implied rental growth rate (Brown & Matysiak, 2000). Geltner and de Neufville (2018) define rental growth as the forecast rate required to project the periodic cash flows of an investment property. Alternatively, it is the constant annual rate of increase in rack rent required to retain the real value of an investmen property and produce an overall rate of return (equated yield) that justifies the exit yield (Baum & Crosby, 2008; Parsons, 2003). Associated with the phenomenon of income growth in property investment is the notion of a standard interval for upward rent review. This rent revision interval is the stipulated period for which rents are expected to be reviewed upward as contained in the lease agreement. According to RICS (1997), rental growth rate may imply an increase or decrease in rent required to forecast rent at a specific date in the future. But in the case of upward rent reviews, it is only rational to have a positive growth rate.

In any typical explicit DCF valuation of leasehold interest or it real value variant, it is rare for the growth rate of cash inflow and cash inflow to be equal otherwise both cash flows would have the same all risks yield which is not possible in practice. Sayce *et al.* (2006) presented a case of complex leasehold interest with non-coinciding reviews for rent received and rent paid. In other words, this phenomenon exerts influence on growth rates of rent received and rent paid respectively. Therefore, if the sales price of a leasehold interest is known, it would be appropriate to use existing scenario analysis techniques to find the optimal growth rates and all risks yield required by the cash flows to produce the market price of a leasehold interest.

Besides the nominal amount quoted as leasehold cash inflow, its associated growth rate relative to that from the cash outflow contributes significantly to leave a surplus of discounted cash flow representing the capital value of the interest. Where the likely purchase price of a leasehold interest has been discerned part-way through rent review epoch, the fundamental questions which this study has put forward to address are:

(a) How can the growth rates of leasehold cash inflow and cash outflow be determined?
(b) what growth rate of leasehold cash inflow relative to cash outflow would be required to achieve this likely purchase price?

Answers to these questions might only be feasible using a blend of leasehold investment valuation models designed to produce identical results. The Implied rental growth rates for cash inflows and cash outflows does not feature at all in the leasehold equivalent yield valuation model so that the only crucial information that can be derived from that model is the price of the leasehold interest. On the other hand, the growth rates being sought after can only be found in the explicit DCF leasehold valuation model. In other words, the equation of the leasehold equivalent yield valuation model (containing the price information) to the explicit DCF leasehold valuation model might likely metamorphose into a hybrid leasehold DCF model from where the all risks yield and growth rates of cash inflows and cash outflows could be iterated and calculated.

Key among the factors driving Investors' sentiments is their rental growth expectation (Clayton *et al.*, 2009), which justifies the need to have a complete hands-on-tools for the assessment of implied rental growth rates for both freehold- and leasehold interests. Previous seminal works by Brown and Matysiak (2000) and Wyatt (2013) specifically used the equivalent yield- and growth explicit DCF valuation models with the aid of the Microsoft® Excel® Solver tool to determine the "single" implied rental growth rate of freehold investment properties valued part-way through a rent review period; but there is a dearth of scholarly efforts addressing the question of how rental growth rates of leasehold investment properties valued part-way through rent review periods could be determined.

This study aims to examine the spreadsheet-assisted scenario analysis tools and techniques that are required for the determination of rental growth rates of leasehold investment properties valued part-way through rent review periods. Objectives put forward to address this aim include setting out an appropriate model for the identification of input parameters for leasehold rental growth determination; setting up a hypothetical case study of reversionary leasehold appraisal; examining the tools and procedure for scenario analysis leading to the determination of appropriate growth rates for leasehold cash inflows and cash outflows; and identifying the required variables to be iterated and calculated within the leasehold DCF valuation scenario.

2. Literature Review And Analytical Framework

The three themes examined in this section comprise leasehold equivalent yield valuation, leasehold growth explicit DCF appraisals, and insights into model-assisted induction of valuation parameters, particularly the iteration of implied rental growth rates of investment properties valued part-way through rent review periods. Shown in Figure 1 is the historical antecedents of models for the valuation of property investments. For the purpose of this study, the review and construction of valuation models were anchored on the assumption that income is received- and payable annually in arrears.

2.1 Equivalent Yield Valuation Technique

Arguably, there are more valuation textbooks and scholarly journals detailing the application of equivalent yield valuation techniques to freehold interests compared to leasehold interests. However, Jones (1983) among other authors at that time availed scholars with the early appearance of an article dealing with equivalent yield valuation technique for leasehold interests using the sinking fund approach which retains a single remunerative rate.

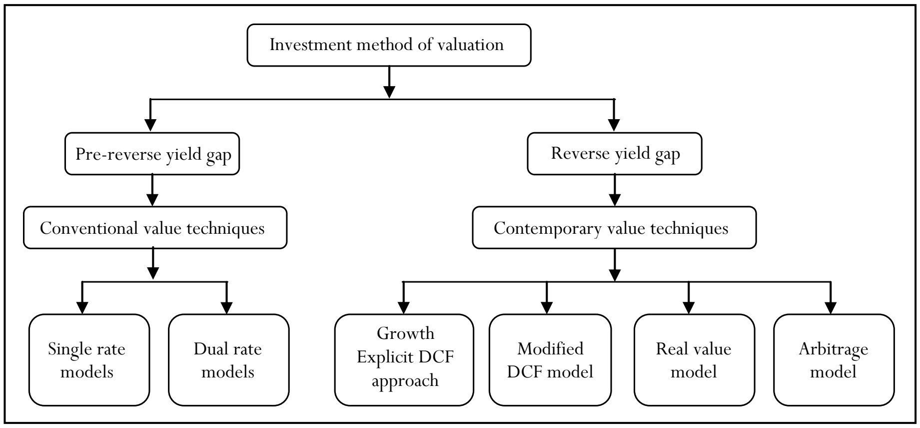


Figure 1 Taxonomy of property investment valuation techniques and associated models

Although Jones (1983) observed nearly four decades ago that the use of equivalent yield in property investment analysis was evolving, the valuation discipline has since evolved academically and professionally by the end of the 20th century and up to the 21st century to warrant awareness of the how this conventional valuation technique can be used to appraise investment properties as noted in seminal works of Brown and Matysiak (2000); Isaac (1998); Sayce *et al.* (2006); Udo (1989), just to mention a few authors. While, Brown and Matysiak (2000); Isaac (2002) and Sayce *et al.* (2006) among others have sustainted the application of equivalent yield valuation techniques to freeholds, Udo (1989) provided insight into how leasehold equivalent yield can be derived upon the completion of explicit DCF valuation of a leasehold interest.

The equivalent yield valuation model for leasehold investments comprises two parts. The first entails calculating the capital value of the contract profit rent in the period to the next rent review, while the second part deals with the capital value of the reversionary profit rent [See Figure 2, and Equations 1 and 2 respectively]. Parameters for the valuation include the equivalent yield, y, the market profit rent, R_1', the contract profit rent, R_0', the number of years to the next rent review, n, and the the unexpired period of reversion to the head lessor, N.

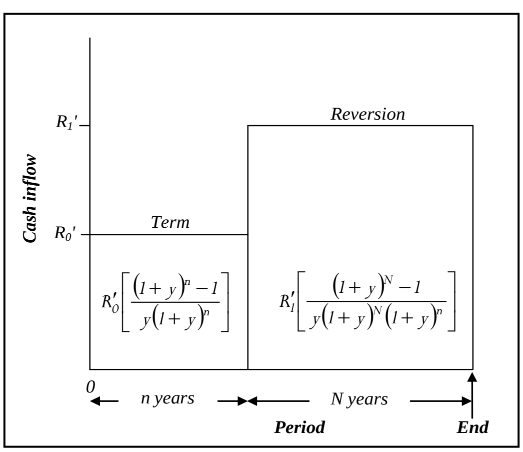


Figure 2: Term and reversion approach to the equivalent yield valuation of leaseholds

Contrary to the layer approach, the term and reversion approach in Figure 2 is deemed logical For leasehold equivalent yield valuation given its terminable characteristic. In other words, leasehold income cannot continue to be earned in perpetuity as construed by the layer approach.

The equation detailing the equivalent yield model of leasehold investment property valuation can be written as:

$$PV = \frac{1}{y(1+y)^n}\left[\left(R_0'\left((1+y)^n-1\right)\right)+\left(\frac{R_1'\left((1+y)^N-1\right)}{(1+y)^N}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Equation 1 can be expressed in the format understood by valuers:

$$PV = \left[\left(\frac{R_0'\left((1+y)^n-1\right)}{y(1+y)^n}\right)+\left(\frac{R_1'\left((1+y)^N-1\right)}{y(1+y)^N(1+y)^n}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Among the advantages of the equivalent yield valuation technique is that it avails valuers with objective analysis of transactions (Baum & Crosby, 2008; Sayce *et al.*, 2006) thereby making it the only conventional technique that can bridge the gap between growth implicit- and growth explicit DCF valuation of reversionary property investments (Brown & Matysiak, 2000). Although there is a dearth of existing literature pertaining to the application of equivalent yield valuation techniques to reversionary leaseholds, surrogates can be found in capital budgeting exercises for terminable investments; see Ajayi (1998); Brown and Matysiak (2000); Dayananda *et al.* (2002); and Luenberger (1998).

2.2 Growth Explicit Investment Valuation Models

With reference to Figure 1, the first of the contemporary value models is the growth explicit DCF approach, which has been demonstrated as the tabulation and discounting of the future values of cash inflows and cash outflows using the Years' Purchase [YP] and Present value [PV] formula at the appropriate nominal rate of interest (equated yield) to arrive at the present values of the cash inflows and cash outflows respectively (Ajayi, 1998; Baum & Crosby, 2008; Baum *et al.*, 2011; Blackledge, 2009; Brown & Matysiak, 2000; Isaac, 2002; Marshall, 1976; Mba, 2020; Sayce *et al.*, 2006; Wyatt, 2013). With respect to this technique, the capital value of the leasehold interest represents the difference between the discounted cash inflow (capital value of rent received) and the discounted cash outflow (capital value of rent paid) (Baum & Crosby, 2008; Butler & Richmond, 1990). The future value of each tranche of cash inflow or cash outflow is determined with recourse to the implied growth rates for rents received and rents paid respectively.

The second category of the contemporary value model is the rational model otherwise called the modified DCF model. When applied strictly to freehold investments, it can be termed the short-cut DCF. In their seminal work, McIntosh and Sykes (1983), Sykes (1984) and McIntosh and Sykes (1985) demonstrated the application of this model to leasehold valuations without recourse to dual rate years purchase. Subsequently, Baum and Yu (1985) provided an improved alternative to the originally developed rational model credited to McIntosh and Sykes (1983) with the intent of handling the valuation of leaseholds with gearing potentials.

For the valuation of rent received (cash inflow), the nominal term rent is discounted at the equated yield over the period to the next rent review and the amount is added to the present value of the growth-adjusted leasehold market rent. In a similar vein, the capital value of rent paid equals the sum of the present value of the nominal head rent passing discounted at the equated yield and the present value of the growth-adjusted market rent payable to the headlessor, so that the capital value of leasehold interest equals the difference between capital value of rent received and the capital value of rent paid.

Equations 3 - 8 in this article provide insights into the structure of the mathematical relationship among the parameters in the rational model that could be used to value a leasehold interest. Although, the rational model was criticised as a re-invented DCF approach (Baum & Crosby, 2008; Crosby, 1986b), it has been demonstrated as an equated yield model, which could be utilized in deriving implied rental growth rates of properties valued part-way through rent review periods (Brown & Matysiak, 2000; Wyatt, 2013).

The third category of the contemporary model is generally called the real value model. Laying the foundation for the evolution of this contemporary value model include Marshall (1976) who developed the "Equated yield model"; Wood (1986a) who developed the "real value model"; and the seminal works of Baum and Crosby (2008), Crosby (1983), Crosby (1984), Crosby (1986c), and Crosby (1986a) which synthesized the equated yield model of Marshall (1976) with the real value model by Wood (1986a) in what was tagged - "the real value/equated yield hybrid model". For the valuation of term income, this technique entails capitalizing the term rent at the equated yield and then adding it to the capital value of the reversion, which is a product of the market rent, a unique variant of income multiplier, and present value of the reversion at the real return (inflation risk free yield) over the period to the next rent review. This unique variant of net income multiplier otherwise called the 3-in-1 Years' Purchase (YP) formula is an incorporation of the nominal rate of interest (equated yield), real return (inflation risk free yield), period to the next rent review, and unexpired term of the investment. Irrespective of the variant of real value model deployed, the capital value of leasehold investment properties remains the difference between the capital value of rent received and that of rent paid.

On the condition that the equated yield, e is greater than the implied rental growth rate, g, the real rate of return (inflation risk free yield), $IRFY$ was found to be mathematical related to the equated yield and implied rental growth rate via the formula:

$$IRFY = \{(1+e)/(1+g)-1\} \quad \ldots\ldots(3)$$

It can be infered from Equation 3 that real value models are basically equated yield models characterized by analytical footprints of alternative data inputs that are implicit about rental growth rates and rent review periods of an investment property.

The fourth contemporary model in Figure 2 is the arbitrage model, which still adopts the convention of term and reversion. The variants of the arbitrage technique include the implicit growth arbitrage technique and the explicit growth arbitrage technique (Crosby, 1996; French & Ward, 1995; French & Ward, 1996). Just as in the equated yield models, the arbitrage growth rate is a function of the low risk yield, R_f; all risk yield, k_0; the full rent review period, t; and the deferred capital yield, DCY becomes:

$$g_{arb} = \frac{1+R_f}{1+\left(\left(\sqrt[t]{\frac{1}{1-\left(k_0 \times Y.P \ for\ t\ years @ R_f\right)}}\right)-1\right)} - 1 \quad \ldots\ldots(4)$$

$$g_{arb} = \{(1+R_f)/(1+DCY)-1\} \quad \ldots\ldots(5)$$

With respect to the explicit arbitrage model, the term rent is valued using the low risk yield, and added to the valuation of the market rent at reversion. The reversionary rent is projected using a different growth rate derived from the low risk yield and deferred capital yield [equation 4] after which it is valued in perpetuity at the all risks yield deferred over the period to the next rent review at the low risk yield (Crosby *et al.*, 1997; French & Ward, 1995; French & Ward, 1996). The explicit arbitrage model could serve as an alternative to the rational model in the iteration and determination of implied rental growth rates of leasehold investment properties valued part-way through rent reviews except that there is dearth of studies regarding an appropriate arbitrage model for reversionary leasehold valuations.

2.3 Techniques of Rental Growth Iteration

The incorporation of rental growth in explicit DCF valuations is informed by the legacy of income growth rate which accrues to recipients of cash inflow and cash outflows respectively. Drawing insights from the seminal works of Adams *et al.* (1999), Doppeigieter and Rode (2002), Fraser (1988), and McGough and Tsolacos (2001), the expression of an asset pricing model is in the form:

$$p_o = \sum_{m=1}^{t} \frac{r_o}{(1+e)^m} + \sum_{m=1}^{t} \frac{r_o(1+g)^t}{(1+e)^{t+m}} + \sum_{m=1}^{t} \frac{r_o(1+g)^{2t}}{(1+e)^{2t+m}} + \ldots\ldots \quad \ldots\ldots(6)$$

where g is the constant rental growth rate per annum; e is the discount rate (equated yield), and t is the period between each rent review, and $e > g$, would imply that the asset price equals the sum of the discounted growth incomes, r_0 over the life of the asset. The simplification of equation 6 becomes:

$$p_o = \frac{r_o}{e - e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1}\right)} \quad \ldots\ldots (7)$$

If all risks yield (capitalization rate), k_0 equals the ratio of cash inflow to the price of an asset then a model of all risks yield similar to that deployed by Fraser (1993) for the pricing of property investments can be expressed as:

$$k_o = e - e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1}\right) \quad \ldots\ldots (8)$$

So that the implied rental growth rate is derived from Equation 8 as:

$$g = \left(\frac{(e-k)(1+e)^t + k}{e}\right)^{1/t} - 1 \quad \ldots\ldots (9)$$

It can be further expressed in the format understood by valuers:

$$(1+g)^t = \frac{YP\ in\ Perp.@\ k - YP\ \ for\ t\ years@\ e}{YP\ in\ Perp.@\ k \times PV\ in\ t\ years@\ e} \quad \ldots\ldots (10)$$

With recourse to Equation 3, the implied rental growth rate, g, could be made the subject of the formula as:

$$g = \{(1+e)/(1+IRFY) - 1\} \quad \ldots\ldots (11)$$

While the theory examined above appears simplistic, the determination of the same rental growth rates for property investments valued part-way through rent review may not really be straightforward.

2.4 Underlying Theory Of "What-If" Scenario Analysis In The Spreadsheet Environment

A spreadsheet software is an application software organized in the form of ledger sheets comprising rows and columns which can be used to perform calculations and alphanumeric operations (Morley & Parker, 2011). Within the context of this article, Microsoft® Excel® 2007 was deployed. Facilitating the deployment of this spreadsheet is the "What-If Analysis" function. Within the context of investment appraisals, "What-if" or scenario analysis has been defined as an experiment designed to unravel the effect of a change in the deterministic value of more than one variable on a single or multiple output variables (Ajayi, 1998; Dayananda *et al.*, 2002). It is an advancement over sensitivity analysis which addresses the impact of a change in one input variable on a single output variable while holding all the other input variables constant. While the outcomes of scenario analysis are commonly expressed in three-folds of base case-, worst case-, and best case scenarios, a different approach involving the concept of slack in linear programming was adopted.

Scenario analysis problems involving constraints are likened to linear programming problems with likely solutions indicating binding or non binding solutions. Cornell (2006) defines slack as a figure representing the difference in the value between the left side (iterated input) constraint and the right side (output) constraint. So that a zero slack implies that the iterated input and the calculated output variables are binding and optimal. In other words, non-binding solutions have values above zero.

The two prominent methods of deploying What-if or scenario analysis in spreadsheets include Goal Seeking and the Solver functions. Cornell (2006) defines goal seeking as the process of finding a single value for a variable in a given cell within the worksheet by changing the value of another associated variable within the worksheet. In other words, goal seeking can only be possible when the existing and unknown variable are related through a system of equations or formula. In MS Excel®, Goal Seek can be instantiated using the command: Data Toolbar > Data tools > What-If Analysis > Goal seek.

An advanced alternative to Goal Seek is the Solver tool. Within the context of Excel®, Cornell (2006) defines the Solver tool as a command that obtains either an exact-, a maximum-, or a minimum value of a worksheet cell by changing other related cells in the same worksheet. Just as in Goal Seeking, the Solver tool can only be instantiated for one or more unknown variables that are related through a system of equations or formula. The Excel® Solver Add-in can be instantiated using the commands: Data Toolbar > Analysis > Solver. Whether the Solver tool or Goal Seek was deployed, the target cell for capital value in the short-cut DCF valuation should be equated with the capital value derived from the "conventional" equivalent yield valuation technique by altering the input cell expected to contain the implied rental growth rate.

Solver- and Goal Seek tools are subsets of What-if analysis in Excel®. Although Goal Seek tool can reference a changing cell and a cell containing a formula across distinct worksheets contrary to the Solver tool, the advantages of the Solver tool over Goal Seek include ability to handle multiple inputs and outputs; ability to determine the minimum, maximum, and exact values of target cell(s); affording user liberty in the specification of constraints and restrictions in cell values; retention of last user settings (Cornell, 2006). The next section examines the previous studies where What-if scenario analysis tool in Excel® among other iteration techniques were deployed in property investment appraisals to determine implied rental growth rates.

2.5 Previous Studies On Complex Rental Growth Iteration

It is recalled from the preceding section that the incorporation of rental growth in explicit DCF valuations was informed by the notion that income growth rate which accrues to recipients of cash inflow and cash outflows respectively. These logical notion appeared in the seminal works of McIntosh and Sykes (1983), McIntosh and Sykes (1985), and Sykes (1984) pertaining the explicit treatment of income growth in both freehold and leasehold appraisals. Besides, other scholarly works featured the simple and complex treatment of rental growth calculation pertaining to investment properties valued part-way through a lease.

In the simplified approach, Fraser (1993) applied equation 8 above to calculate rental growth rate. Similarly, Isaac (2002) applied equation 8 and further suggested equation 10 as surrogate. A variant of equation 9 was deployed by Baum *et al.* (2011) and French (2006) to calculate implied rental growth rate. The undelying gap in the studies of rental growth calculations was the illustrative application to freehold investment property while ignoring the treatment of leasehold investment properties.

Since implied rental growth rate is primarily featured in explicit DCF appraisal models, it may be recalled from Sayce *et al.* (2006) that the investment value of leaseholds equals the difference between discounted cash inflow (rent received) and discounted cash outflow (rent paid) so that a complicated problem of having to iterate and determine growth rates for both cash flows is created. Hence, the simple deterministic models in equations 9 - 11 is deemed inappropriate to handle such complex situations.

Although Wyatt (2013) acknowledged the existence of the problem of determining rental growth rate across rent review period, he only provided brief notes on the solution to such a challenge without specifically mentioning spreadsheet or iteration tool that can be used to address the problem.

Within the context of freehold investment property, Brown and Matysiak (2000) demonstrated three techniques for the calculation of implied rental growth rate part-way through rent review epochs using available data comprising price, rent passing, market rent, number of years to the next rent review, equivalent yield, and the nominal yield. These techniques include graphical solutions, What-if scenario analysis tool - Excel® Solver, and the development and use of an Excel® Add-in called "RVGrowth" using Visual BASIC Programming.

According to Brown and Matysiak (2000), the graphical solution to implied rental growth rate calculation entails finding the point of intersection at the abscissa where the simultaneous algebraic equations relating to the equivalent yield and periodic growth intersects.

The second is a software iteration technique involving the What-if scenario analysis tool comprising Excel® Solver Add-in and the Excel® Goal Seek commands. Although the Excel® Goal Seek approach appears to be simpler than the Excel® Solver and Graphical solutions, Brown and Matysiak (2000) did not demonstrate its use at that time. Nevertheless, a limitation of the Goal Seek to the appraisal of income growth rates in leasehold investments is that it can only address growth rate of cash inflow or cash outflow at a time whereas, Excel® Solver can address iteration of multiple variables (Cornell, 2006); hence its adaptability to simultaenous rental growth rate iterations for cash inflow and cash outflows respectively.

The third technique demonstrated by Brown and Matysiak (2000) to calculate freehold rental growth rate was the deployment of Visual BASIC Programming to write and compile a program that computes rental value growth across rent review epochs and then deploy same program as Excel® Add-in. While applauding this approach as an unprecedented feet in the interface between spreadsheet iteration and property appraisal, its application to leasehold investment property is beyond the scope of this study and has been reserved for further studies. This is because the use of this technique to determine optimal growth rate of leasehold cash flows is not really straightforward since input parameters in leasehold equivalent yield valuation- and the growth explicit leasehold DCF valuation models would have to be iterated to determine two growth rates namely - the growth rate for rent received and the growth rate for rent payable. In this view, it is important to point out that this study should not be construed to be synonymous to the seminal work of Nanthakumaran (1988) who examined the application of the two growth rate model to the appraisal of leaseholds. The focus is on determining the simultaneous values of rental growth rates for cash inflows and cash outflows required to produce an indicated price of an interest in leasehold investment property.

2.6 Impact Of Embedded Options In Leases

In financial parlance, an option is defined as "*the right without obligation to obtain something of value upon the payment or giving up of something else of value*" (Geltner *et al.*, 2010). There are real options that coul be associated with real estate investing. Pezeshkian *et al.* (2014) outlined some of them to include option to purchase land for development, option to renew a lease and the option to terminate a lease. Others include option to invest (Lucius, 2001); option to sescure debt or equity finance (Shen & Pretorius, 2013); option of upward-only rent reviews and option of upward- and downward rent reviews (Ward & French, 1997); option to purchase or lease (Hargitay & Yu, 1993); break clauses, option for change of use (Booth *et al.*, 2001); and option of pre-emption rights (Buetow & Albert, 1998). An investor instantiates an option when (s)he exercises a right to invest in landed property at a future date of his or her choice. An option is a right and not an obligation to invest (Lucius, 2001). The determination of capital value of an interest in real property in anchored on the process of ascertaining the present value of the right to receive a stream of annuity subject at a discount rate over a definite or indeterminate period of time; so that in the investment valuation context, an option is

exercised by an investor who is entitled to a right to receive the sum of discounted income streams over a given period of time.
Among the array of seminal works on option pricing of real estate decisions, an attempt shall be made to examine five related studies. First, Ward and French (1997) deployed a combination of arbitrage valuation and the Black and Scholes model and concluded that the loss of option to restrain upward-only rent reviews can significantly improve the attractiveness of property investment.

With respect the pricing of embedded options in a lease, Buetow and Albert (1998) developed a partial differential equation that describes option granted to a lessee to either purchase a leased property or to renew the lease at a price at par with the consumer price index (CPI). By implication, it is possible for a lease contract to avail a lessee the alternatives of either exercising pre-emption right of purchasing her landlord's property or lease renewal upon the payment of an upward-reviewed rent if the termination of the contractual agreement may not be favourable to both parties.

Lucius (2001) provided a critique of the application of option pricing theory to investment valuation and concluded that the pricing of options in real estate decision-making is characterized by academically abstract results which have limited practical applications to real estate projects. Lucius (2001) at that time however suggested the conduct of further research aimed at fostering a transition of that body of knowledge into the practice of property investment valuation.

With respect to property development, Booth *et al.* (2001) underscored the possibility of risk arising from changes in occupier market condition, delays in project completion and cost overruns which are injurious to project viability. In the first instance, adverse changes in occupier market condition and delayed completion might exercebate rental loss. Secondly, whether or not there is rental loss, these risks are embedded in the rents payable by the leasehold investor and the sub-lessee in occupation of a leasehold property so that adequate measures for maintenance of optimal profit rent margin may arise.

Adams *et al.* (2003) acknowledged the deficiency of DCF valuation techniques to handle array of options contained in lease contracts and made a compelling case for the development of techniques from the field of finance to handle complex lease contracts and options. While applauding the strength of the option pricing technique over DCF valuation technique, Adams *et al.* (2003) observed that the valuation of lease options was yet to be accorded any significant attention in practice, which aligns with a similar observation by Lucius (2001).

With respect to the use of binomial option pricing technique to assess the portfolio value of real estate developments, Shen and Pretorius (2013) found that timely completion of projects would curtail cost overrun and avail the developer with excess capital for further projects. According to Shen and Pretorius (2013), the application of real option valuation in property development practice is determined by the structure and available resources of the developer.

Pezeshkian *et al.* (2014) demonstrated the deployment of tools from the field of finance and decision trees to address the pricing of real estate options comprising land purchase option, lease renewal option, and lease termination option using industry-related case studies. They however did not mention the extent to which the option pricing technique is applicable to real estate market particularly in Florida and the United States in general.

Turning attention to the synergy between rental growth analysis and option pricing associated with loss of rent, it could be recalled that the valuation of reversionary leasehold interest is made up of the valuation of term and reversionary cash inflows and cash outflows respectively. The actual rents received and payable are laid out in the term, while the anticipated or market rents receivable and payable are laid out in the reversion.

First, actual rental loss in property development could arise from period of voids that precedes letting. So that a longer period of void will exercebate a higher risk of loss in the actual rent. Secondly, the risk of tenant or sub-tenant default also contributes to the phenomenon of actual rental loss in the sense that the present value of a rent payable after some periods of default is diminshed unless specific clauses have been introduced in the tenancy agreement to demand a percentage of the owed rent as penalty to shield the rent payable against inflation and real value dimunition.

On the other hand, the loss of market rent might arise from two phenomena. The first is a situation where an income producing property is let at a rent below the market rental value. The second is attributed to abnormal timing of contract rent revision beyond the market rent of comparable properties. The explicit DCF valuation technique has been demonstrated to handle similar situation of rental loss especially in the case of over rented properties where contract rent exceeds the market rent (Adams & Booth, 1996; Crosby, 1996; Crosby & Goodchild, 1993; Crosby & Henneberry, 2016); however Adams and Booth (1996) suggested the deployment of sophisticated appraisal techniques to surmount the existing deficiencies of the existing DCF techniques.

With these practical realities, what should real estate investors or developers generally do when confronted with the problem of rental loss? In consonance with the seminal work of Ward and French (1997), it is possible to envisage that the attractiveness of property investment is enhanced significantly when investors exercise measures aimed at ameliorating rental loss. For the developer, this may be achieved through income gains associated with timely completion of a project (Shen & Pretorius, 2013). Notwithstanding this measure, the developer is bound to face rental value loss during the construction phase and would have to "wait for more time to compensate for this loss" (Shen & Pretorius, 2013).

It is pertinent at this juncture to mention that the possibility of rental loss in leasehold interest is acknowledged in this study from an option pricing perspective. However, it does not form part of the scope of this research which has the aim of using spreadsheet-embedded scenario analysis tools to determine growth rates of cash inflows and cash outflows of reversionary leaseholds valued part-way through rent review epochs.

2.7 Analytical Framework For Leasehold Rental Growth Iteration

A typical diagram of stepped leasehold rent in Figure 3 analyzes the treatment of growth-induced cash inflow and outflow arising from upward rent revision at stipulated epochs. At the start of the lease in year 0, the leaseholder earns a contract rent, R_0 from the sublease. The total holding period available to the leaseholder at first grant from the freeholder is "$3t$" years. It is further observed that cash inflow is revised upward on a t-yearly basis. On the valuation date, V_d; the number of years to the next revision of rent receivable is (t - V_d) years, which would eventually be used in conjunction with a nominal rate of interest (equated yield) to determine the value of the term. Again at the valuation date, V_d; the leasehold market rent remains static at R_1 until the next upward review. The future value of R_1 at the first revision is $R_1'(1+g)^t$, which is equivalent to $R_1(1+g)^{t-Vd}$. This rent increases to $R_1'(1+g)^{2t}$ and $R_1'(1+g)^{3t}$ during the second and third periods of upward reviews to align with the numerators of each tranche of discounted cash flow in equation 6. The phenomenon in Figure 3 equally applies to the determination of the present value of rent payable.

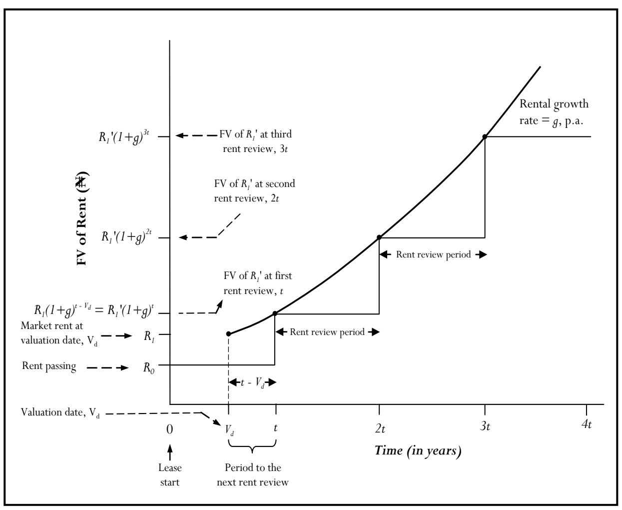


Figure 3 Term- and future rent review profile of leaseholds

The stepped rents in Figure 3 is only a simplification of the reality. Figure 4 depicts the actual nature of the stepped rents at each rent review epoch.

In actual sense, the transition of future rent across each review date is not regular as shown in Figure 4 so that the fitted exponential trend line of the form $FV = ae^{b(time)}$ would help explain the rising profile of rent paid or rent received throughout the life of a leasehold investment. If Figures 3 and 4 would apply to the cases of rent received and rent payable by the leaseholder, then the growth rates of rent received and rent payable can be tagged as g_0 and g_1 respectively.

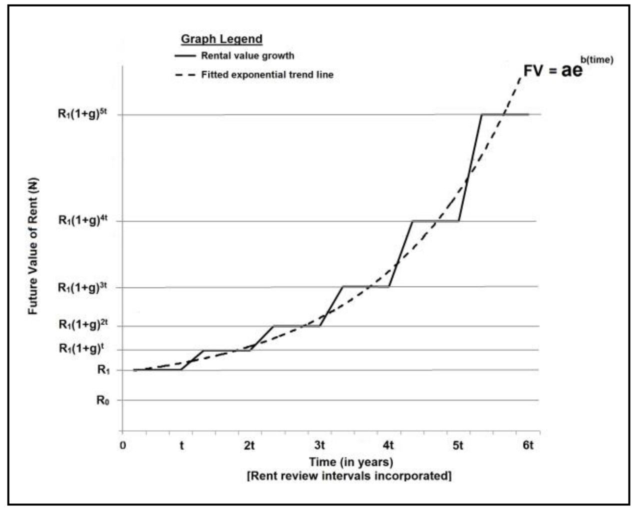


Figure 4 Actual nature of term- and future rent review profile

In order to determine rental growth rate of a freehold, Brown and Matysiak (2000) and Wyatt (2013) set up a system of equation that placed the freehold equivalent yield valuation model on the left hand side, and the freehold DCF valuation model with rent review period and growth on the right hand side as shown in equation 12 without using the Y.P. notation:

$$\left[\frac{R_0}{y_0}+\frac{R_1-R_0}{y_0(1+y_0)^n}\right]=\frac{R_0\{(1+e)^n-1\}}{e(1+e)^n}+\frac{R_1(1+g)^n}{e(1+e)^n}\left[\frac{(1+e)^t-1}{(1+e)^t-(1+g)^t}\right] \ldots\ldots (12)$$

Whether or not the all risks yield of an investment property is known, a surrogate for the Years purchase in Perpetuity is derived from equation 12 as:

$$\frac{1}{k_0}=\left[\frac{(1+e)^t-1}{e\left((1+e)^t-(1+g)^t\right)}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Therefore, equation 12 can be reduced to the format in equaton 14 as:

$$\left[\frac{R_0}{y_0}+\frac{R_1-R_0}{y_0(1+y_0)^n}\right]=\left[\frac{R_0\{(1+e)^n-1\}}{e(1+e)^n}+\frac{R_1(1+g)^n}{k_0(1+e)^n}\right] \ldots\ldots\ldots\ldots (14)$$

Equated yield however becomes the capitalization rate for stationary incomes in the valuation of the term cash inflows; that is $k_0 = e$, (Ifediora, 2005).

This study examines a case of Reversionary leasehold where rent paid grows throughout the term coupled with a reversion of rent payable; in which case, two growth rates comprising the growth rate for rent received, g_0 and that for rent payable $g_1$would have to be calculated. The appropriate growth explicit DCF model for the valuation of this instance of leasehold interest is captured in Equation 15 where, e = equated yield, R_0 = rent received by the leaseholder, R_1 = leasehold market rent, g_0 = implied rental growth rate of leasehold income, k_0 = All risks yield of leasehold income, $n = (t - V_d)$ = number of years to the next leasehold rent review, N = unexpired term of leasehold investment, r_0 = rent paid to the freeholder, r_1 = freehold rack (revised) rent g_1 = implied growth rate of rent paid, k_1 = All risks yield of freehold income, and m = number of years to the next freehold rent review.

In the event where the full reversion of a leasehold interest coincides with a full reversion of rent payable, equations 15 and 16 could be trimmed to equations 17 and 18. Equation 15 was used to perform the spreadsheet iteration, while the presentation of valuation scenarios was carried using equations 15 and 17 where applicable. The spreadsheet iteration was supported by equating the models captured in equations 2 and 15 respectively.

$$\text{PV of interest} = \left[\left(R_0\left(\frac{1-(1+e)^{-n}}{e}\right)\right)+\left(\frac{R_1}{k_0}\left(\left(\frac{(1+g_0)^n}{(1+e)^n}\right)-\left(\frac{(1+g_0)^N}{(1+e)^N}\right)\right)\right)\right]-\left[\left(r_0\left(\frac{1-(1+e)^{-m}}{e}\right)\right)+\left(\frac{r_1}{k_1}\left(\left(\frac{(1+g_1)^m}{(1+e)^m}\right)-\left(\frac{(1+g_1)^N}{(1+e)^N}\right)\right)\right)\right] \quad \text{(15)}$$

Equation 14 can be written conventionally as:

PV = [R_0(Y.P. for n years @ e) + R_1(Y.P. for N years @ k_0 deferred for n years @ e] - [r_0(Y.P. for m years @ e) + r_1(Y.P. for N years @ k_1 deferred for m years @ e] .... (16)

On the other hand, leasehold valuation model where full reversion of a leasehold interest coincides with a full reversion of rent payable:

$$\text{PV of interest} = \left(\frac{R_1}{k_0}\left(\left(\frac{(1+g_0)^n}{(1+e)^n}\right)-\left(\frac{(1+g_0)^N}{(1+e)^N}\right)\right)\right) - \left(\frac{r_1}{k_1}\left(\left(\frac{(1+g_1)^m}{(1+e)^m}\right)-\left(\frac{(1+g_1)^N}{(1+e)^N}\right)\right)\right) \quad \text{(17)}$$

Or conventionally expressed as:

PV = [R_1(Y.P. for N years @ k_0 deferred for n years @ e] - [r_1(Y.P. for N years @ k_1 deferred for m years @ e] .... (18)

where all the parameters in equations 17 and 18 maintain the same meaning as described in equations 15 and 16.

Placing the leasehold growth explicit DCF valuation model [Equation 15] on the left hand side and the leasehold equivalent yield valuation model [Equation 2] on right hand side informed the deployment of an appropriate layout [Figure 6] of the data inputs for the appraisal and rental growth iterations.

$$\left[\left(\boldsymbol{R_0}\left(\frac{(1-(1+e))^{-n}}{e}\right)\right)+\frac{\boldsymbol{R_1}}{k_0}\left(\left(\frac{(1+g_0)^n}{(1+e)^n}\right)-\left(\frac{(1+g_0)^N}{(1+e)^N}\right)\right)\right]-\left[\left(\boldsymbol{r_0}\left(\frac{(1-(1+e))^{-m}}{e}\right)\right)+\frac{\boldsymbol{r_1}}{k_1}\left(\left(\frac{(1+g_1)^m}{(1+e)^m}\right)-\left(\frac{(1+g_1)^N}{(1+e)^N}\right)\right)\right]=\left[\left(\boldsymbol{R_0'}\left(\frac{(1-(1+y))^{-n}}{y}\right)\right)+R_1'\left(\frac{(1-(1+y))^{-N}}{y(1+y)^n}\right)\right] \quad \text{(19)}$$

Contrary to the procedure for the determination of a "single" implied rental growth rate for freehold investment, Equation 19 was used to help address the question of what growth rates in cash inflow and cash outflow would be required for a leasehold interest to achieve a desired purchase price.

3. Methodology

3.1 Data Requirements

For the purpose of this experimental research design, data required for the modelling of growth rate of leasehold cash inflow and cash outflow were drawn from the valuation case study involving a reversionary leasehold investment property. Details are shown in Table 1.

3.2 Valuation Case Study

The contemporary valuation problem used to illustrate Spreadsheet iteration of Leasehold rental growth rate is a case where the holder of the leasehold interest obtained consent from the freeholder 2 years ago to sublet a commercial property for a term of 20 years at a contract rent of ₦1,200,000 per annum subject to 5-yearly upward review. The market rent accruing to the leaseholder was determined to be ₦1,500,000 per annum and subject to 5-yearly upward review. The leasehold interest in question was secured from the freeholder at a head rent of ₦180,000 per annum reviewed at 3-yearly interval. While this head rent paid shall be revised upward in 2 years' time, the market head rent payable is put at ₦250,0000 per annum reviewable at 3-yearly interval. Given an equated yield of 25%, the leasehold interest may likely be purchased today at a price of ₦8,704,728.55. The valuation layout leading to the determination of the leasehold and freehold rental growth rates necessary to achieve the purchase price of ₦8,704,728.55 was prepared in Excel® as indicated in Figure 5.

3.3 Software Specification And Application Tools

While any reasonable computer hardware (desktop or notebook) could be used to perform the operations leading to valuation scenario analysis, this section emphasizes on the preparation and use of MS Excel in the performance of spreadsheet iterations required to address the research problem.

A1 fx Valuation data

	A	B	C	D	E	F	G	H	I	J
1	**Valuation data**									
2	Equated yield	25%								
3	Leasehold Equivalent yield, ke	11.63717960%								
4	Leasehold rent received	1,200,000								
5	Leasehold market rent	1,500,000								
6	Leasehold rent review	5	yearly							
7	Leasehold rental growth, g0	0.00000000%								
8	All risks yield of leasehold income, k0	25.00000000%								
9	Rent paid to the freeholder	180,000								
10	Revised head rent	250,000								
11	Freehold rent review	3	yearly							
12	freehold rental growth, g1	0.00000000%								
13	All risks yield of freehold income, k1	25.00000%								
14	Number of years to the next leasehold rent review	3	years							
15	Number of years to the next freehold rent review	2	years							
16	Unexpired term of leasehold interest	18	years							
17										
18										
19	**Equivalent yield Valuation**									
20	Term									
21	Rent received						1,200,000			
22	Rent paid						180,000			
23	Profit rent						1,020,000			
24	Y.P. for 3 years @ ke						2.416885653		2,465,223.37	
25										
26	Reversion									
27	Leasehold market rent						1,500,000			
28	Revised rent payable						250,000			
29	Revised profit rent						1,250,000			
30	Y.P. for 15 years @ ke				6.9449					
31	Less P.V. of ₦1 in 3 years @ ke				0.7187		4.991604147		6,239,505.18	
32	**Valuation**								8,704,728.55	
33										
34										
35										
36	**Growth Explicit DCF Valuation**									
37	Term									
38	Rent received				1,200,000					
39	Y.P. for 3 years @ 25%				1.9520		2,342,400.00			
40										
41	Reversion									
42	Leasehold market rent				1,500,000					
43	Amount of ₦1 in 3 years @ g0	1.0000								
44	Y.P. in Perp @ k0	4.0000								
45	P.V of ₦1 in 3 years @ 25%	0.5120	2.0480							
46										
47	Less Amount of ₦1 in 18 years @ g0	1.0000								
48	Y.P. in Perp @ k0	4.0000								
49	P.V of ₦1 in 18 years @ 25%	0.0180	0.0721		1.9759		2,963,913.61		5,306,313.61	
50										
51	Rent paid				180,000					
52	Y.P. for 2 years @ 25%				1.4400		259,200.00			
53										
54	Reversion									
55	Leasehold market rent				250,000					
56	Amount of ₦1 in 2 years @ g1	1.0000								
57	Y.P. in Perp @ k1	4.0000								
58	P.V of ₦1 in 2 years @ 25%	0.6400	2.5600							
59										
60	Less Amount of ₦1 in 18 years @ g1	1.0000								
61	Y.P. in Perp @ k1	4.0000								
62	P.V of ₦1 in 18 years @ 25%	0.0180	0.0721		2.4879		621,985.60		881,185.60	
63	**Present Value of Leasehold interest**								4,425,128.01	
64										

Figure 5 Valuation framework for the iteration of all risks yield and implied rental growth rates

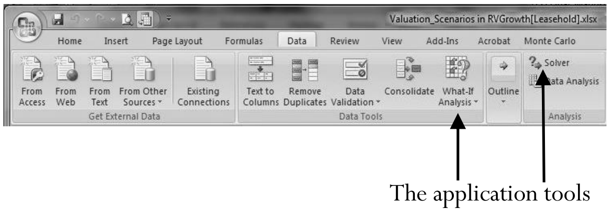


Figure 6 Tools deployed for the scenario analysis

From the Menu Bar tagged "Data", the two tool bars specified for this study are the What-If Analysis data tool and the analysis tool tagged "Solver" (See Figure 6).

Cells "B8" and "B13" in Figure 5 where initially edited with equation 20 for the purpose of computing all risks yield of cash inflow and cash outflow respectively. These yields represent the implicit return on capital value of cash inflow and cash outflow respectively. By virtue of the spreadsheet design in Figure 6, the optimal all risks yield for cash inflow and cash outflow were automatically calculated when the optimal implied rental growth rates for cash inflow and cash outflow were returned in cells "B7" and "B12" following the conclusion of the iteration process using Excel Solver tool.

With recourse to Figure 1 above, the What-If tool was deployed to run the Goal Seek function for the determination of equivalent yield (See Table 2), while the Solver tool under the Analysis tool bar was used to perform the iteration and determination of rental growth rates of the leasehold investment property featured in the case study (See Figure 7).

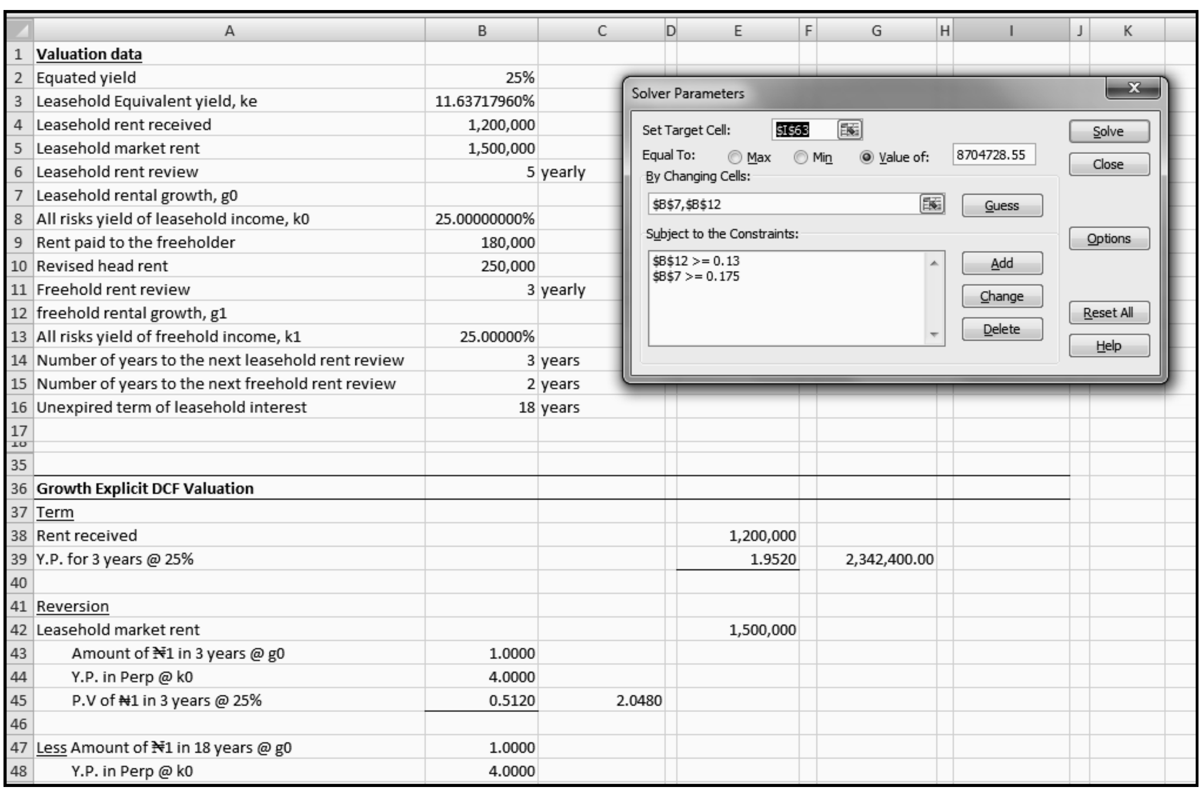
The abridged format of the DCF table used to determine the leasehold equivalent yield was presented in Table 2. Equation 9 was used to calculate the implied rental growth rate. Furthermore, on the condition that $g < e$, the formula deployed in MS Excel® to calculate the all risks yield of leasehold and freehold incomes is generally expressed as the reciprocal of Equation 13:

$$k_0 = \frac{e\left((1+e)^t - (1+g)^t\right)}{(1+e)^t - 1} \quad \text{.......} \quad (20)$$

Both parameters of g and k were computed simultaneously using the Solver function. The contents of the Equivalent yield valuation and the growth explicit DCF valuation were numerically linked to the valuation data in Cells "B1" to "B16".

Prior to the commencement of the scenario analysis, the iterated values for the leasehold rental growth rate were accorded a range of 14.0% to 19.0% and at an interval of 0.5%; while the iterated growth rates of rent payable were accorded a range of 9.5% to 14.0% and at the same interval of 0.5%. This proposition was anchored on the possibility that the true growth rates would fall within these range of values.

As mentioned in section 2.4, the appropriate scenario analysis tool was instantiated with the operation of Data menu button as follows: Data > Analysis > Solver.



	A	B	C	D	E	F	G
1	Valuation data						
2	Equated yield	25%					
3	Leasehold Equivalent yield, ke	11.63717960%					
4	Leasehold rent received	1,200,000					
5	Leasehold market rent	1,500,000					
6	Leasehold rent review	5	yearly				
7	Leasehold rental growth, g0						
8	All risks yield of leasehold income, k0	25.00000000%					
9	Rent paid to the freeholder	180,000					
10	Revised head rent	250,000					
11	Freehold rent review	3	yearly				
12	freehold rental growth, g1						
13	All risks yield of freehold income, k1	25.00000%					
14	Number of years to the next leasehold rent review	3	years				
15	Number of years to the next freehold rent review	2	years				
16	Unexpired term of leasehold interest	18	years				
17							
35							
36	Growth Explicit DCF Valuation						
37	Term						
38	Rent received				1,200,000		
39	Y.P. for 3 years @ 25%				1.9520		2,342,400.00
40							
41	Reversion						
42	Leasehold market rent				1,500,000		
43	Amount of ₦1 in 3 years @ g0	1.0000					
44	Y.P. in Perp @ k0	4.0000					
45	P.V of ₦1 in 3 years @ 25%	0.5120	2.0480				
46							
47	Less Amount of ₦1 in 18 years @ g0	1.0000					
48	Y.P. in Perp @ k0	4.0000					

Figure 7 Scenario Analysis of leasehold and freehold rental growth rates using What-If Solver function in MS Excel

A dialogue box appeared as shown in Figure 7. The Solver tool was instantiated to carryout scenario analysis of leasehold capital values vis-á-vis the implied growth rates for rent received and rent payable. Cell "I63" is the target cell for the calculation of the leasehold capital value in the growth explicit model. Within the solver dialogue box, the cell "I63" was set to an instant value of 8704728.55 by changing the cells B7 and B12 representing the leasehold and freehold rental growth rates respectively. The best approach was to set these iterations as constraints using the "greater than or equal to" sign. The solver engine was prompted through the dialogue box in Figure 7 to conduct 100 iterations by default and return results with 0.000001 precision in a maximum of 100 seconds. Other selected meta analysis for the scenario analysis include Quadratic estimates, Central Derivatives, and the embedded use of Newton's method for solving numerical equations derived from the explicit discounted cash flow technique. The central derivative box was selected to help control rapid divergence between iterated and calculated growth rates.

3.4 Decision Rule For The Scenario Analyses

Associated with each scenario of cash inflow and cash outflow is the **tabulation** of growth rate constraint, calculated growth rates, slack in the calculated growth rates, and present values leading to the capital value of the leasehold interest (See Figure 8). Within this context, a slack is defined as the numerical difference between iterated and calculated values of a given parameter; so that the **decision rule** for the scenario analysis is to accept the iterated and calculated growth rates of cash inflows and cash outflows that simultaneously exhibit zero slacks.

Therefore, **inference** could be drawn regarding the true growth rates as those that simultaneously return zero slacks for iterated and calculated growth rates of leasehold discounted cash inflows and cash outflows respectively.

4. Scenario Analysis And Data Presentation

4.1 Preliminary Data

Table 1 indicates the preliminary valuation data associated with the valuation case study in section 3.2. It is observed from Table 1 that the valuation problem is silent on the growth rates of rent received and rent paid respectively. In order to deploy these data for the spreadsheet iteration and determine these growth rates, the all risks yields of leasehold and freehold incomes, and the leasehold equivalent yield shall be computed. Computation of these parameters is anchored on the fact that they are embedded in the valuation problem and could be extracted using the techniques described in the preceding section.

4.2 Goal Seek Calculation Of Equivalent Yield

With recourse to the equivalent yield valuation model, the profit rents for the leasehold interest under consideration are put at ₦1,020,000 for the term and ₦1,250,000 at reversion. With the likely leasehold purchase price of ₦8,704,728.55, the Goal seek function of MS Excel's What-If Analysis tool was deployed to determine the leasehold equivalent yield as 11.63717960%. The abridged DCF for the determination of this equivalent yield is indicated in Table 2.The result of the What-If Analysis indicates that it would take a leasehold equivalent yield of 11.63717960% to achieve a likely purchase price of ₦8,704,728.55 in the market.

The question now would be - At what rental growth rates would the leaseholder realize a capital value of ₦8,704,728.55 for the commercial property in question? Full data specification and base case valuation shown in Figure 5 was set up to help provide a feasible answer.

Table 1 Valuation data for rental growth determination

Equated yield, e	25%
Leasehold Equivalent yield, k_e	?
Leasehold rent received	₦1,200,000
Leasehold market rent	₦1,500,000
Leasehold rent review	5 yearly
Leasehold rental growth, g_0	?
All risks yield of leasehold income, k_0	?
Rent paid to the freeholder	₦180,000
Revised head rent	₦250,000
Freehold rent review	3 yearly
freehold rental growth, g_1	?
All risks yield of freehold income, k_1	?
Number of years to the next leasehold rent review	3 years
Number of years to the next freehold rent review	2 years
Unexpired term of leasehold interest	18 years
Likely purchase price of Leasehold interest (today)	₦8,704,728.55

Table 2 Abridged Discounted Cash Flow technique of equivalent yield determination

Purchase price of property:		₦8,704,728.55		
Target rate of return, R_x:		11.63717960%		
Term:		3 years		
Reversion:		15 years		
Year	**Cash inflow**	**Y.P for 15 years @ R_x%**	**P.V. of ₦1 @ R_x%**	**Present Value**
1	1,020,000		0.895758925	913,674.10
2	1,020,000		0.802384052	818,431.73
3	1,020,000		0.718742676	733,117.53
4 - 18	1,250,000	6.944911323	0.718742676	6,239,505.19
	Present value of cash inflow @ R_x% ………………………………			8,704,728.55
<u>Less</u>	Purchase price of property ………………………………………			8,704,728.55
	Net Present Value ……………………………………………			**0.00**

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Parameters**	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**	**Scenario 6**	**Scenario 7**	**Scenario 8**	**Scenario 9**	**Scenario 10**	**Scenario 11**
2	**Leasehold rental growth constraint**	≥ 14.0%	≥ 14.5%	≥ 15.0%	≥ 15.5%	≥ 16.0%	≥ 16.5%	≥ 17.0%	≥ 17.5%	≥ 18.0%	≥ 18.5%	≥ 19.0%
3	**Freehold rental growth constraint**	≥ 9.5%	≥ 10.0%	≥ 10.5%	≥ 11.0%	≥ 11.5%	≥ 12.0%	≥ 12.5%	≥ 13.0%	≥ 13.0%	≥ 13.5%	≥ 14.0%
4	**Calculated Leasehold rental growth**	17.5277%	17.5890%	17.6522%	17.7175%	17.7849%	17.8543%	17.9260%	18.0000%	18.0000%	18.5000%	19.0000%
5	**Calculated Freehold rental growth**	9.5000%	10.0000%	10.5000%	11.0000%	11.5000%	12.0000%	12.5000%	13.0000%	13.0000%	16.0218%	18.5630%
6	**Slack in calculated Leasehold rental growth**	3.5277%	3.0890%	2.6522%	2.2175%	1.7849%	1.3543%	0.9260%	0.5000%	0.0000%	0.0000%	0.0000%
7	**Slack in calculated Freehold rental growth**	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	2.5218%	4.5630%
8	**PV of Leasehold rent received**	9,969,028.69	9,996,898.40	10,025,776.26	10,055,704.67	10,086,727.86	10,118,891.99	10,152,245.22	10,186,837.83	10,186,837.83	10,425,444.79	10,672,639.24
9	**PV of rent paid**	1,264,300.14	1,292,169.85	1,321,047.71	1,350,976.12	1,381,999.31	1,414,163.44	1,447,516.67	1,482,109.28	1,482,109.28	1,720,716.24	1,967,910.69
10	**Capital value of Leasehold interest**	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55	8,704,728.55

Figure 8 Scenarios of leasehold and freehold rental growth rates leading to ₦8,704,728.55 leasehold capital value

4.3 *Scenario Analysis Using The Solver Tool*

Eleven valuation scenarios were generated [Figure 8]. It can be observed that all the scenarios of varying leasehold and freehold rental growth rates returned leasehold capital value of ₦8,704,728.55 after holding the values of all other input parameters in Table 1 constant. This capital value represents the difference between the present value of leasehold rent received and the present value of rent paid.

For scenarios 1 to 7, slacks in the neighbourhood of 3.5277% and 0.9260% were returned for calculated leasehold rental growth rates following the deployment of iterated growth rates in the range of 14.0% to 17.0%. There was no corresponding slack in the calculated freehold rental growth rates for each scenario of iterated growth rates in the range of 9.5% to 12.5%. The implication of these simultaneous results of slack in leasehold rental growth rate and zero slack in freehold rental growth rates is that the results violated the decision rule for the appropriate income growth rates for leasehold investment properties valued part-way through a rent review period.

Drawing attention to the eighth scenario, the iterated leasehold rental growth rate of 17.5% returned a calculated growth rate of 18.0%, representing 0.5% slack. On the other hand, the iterated freehold rental growth rate of 13.0% returned a calculated growth rate of 13.0%, representing zero slack. Consequently, the real-time DCF valuation of the leasehold interest deployed the calculated leasehold income growth rate of 18.0% to return ₦10,186,837.83 as present value of cash inflow, while the freehold income growth rate of 13.0% returned ₦1,482,109.28 as the present value of cash outflow. In other words, the eighth scenario analysis provided insight into the true growth rates for cash inflow and cash outflow as 18% and 13% respectively.

Consequently, the ninth scenario which is a deviation from the 0.5% interval earlier specified returned calculated growth rates of 18.0% and 13.0% for leasehold cash inflow and cash outflow for the 18.0% and 13.0% iterated growth rates; implying a zero slack and a validation of the decision rule for the appropriate income growth rates for leasehold investment properties valued part-way through a rent review period.

Scenario results from the tenth and eleventh iteration indicated slacks which diminished the reliability of the calculated rental growth rates and violated the decision rule for the appropriate income growth rates for leasehold investment properties valued part-way through a rent review period.

Inference can be drawn that the ninth scenario presents the optimal growth rates for cash inflow and cash outflow required by the investor to realize the likely purchase price of ₦8,704,728.55.

4.4 *Validation of Rental Growth Rates*

Attempts were made in this section to carryout array of appraisals aimed at validating the growth rates obtained from the 9th scenario. In Table 3, the summary of the results from the ninth scenario analysis indicates that the leasehold investor might require 18% and 13% growth rates in cash inflow and cash outflow to realize a likely purchase price of ₦8,704,728.55.

Where k_0 = 9.30909159% and k_1 = 13.38303% and the leasehold- and freehold implied rental growth rates are fixed at

18% and 13% respectively, results of the remaining three scenario-generated valuations in Table 4, Table 5 and Table 6 indicated an increase (decrease) in the leasehold equivalent yield (capital value) from 11.63717960% (₦8,704,728.55) in the 3rd year to 13.51423732% (₦7,867,964.66) at full reversion of the sublease to the leaseholder.

Results from the conduct of scenario tests across the appraisals in Table 4, table 5 and table 6 indicated zero slack between the iterated and calculated leasehold- and freehold rental growth rates of 18% and 13% respectively.

Table 3: Value parameters at 3 years to the next rent review

Parameter	Value
Equivalent yield valuation	₦8,704,728.55
Explicit DCF valuation	₦8,704,728.55
Leasehold Equivalent yield, k_e	11.63717960%
All risks yield of leasehold income, k_0	9.30909159%
Leasehold rental growth, g_0	18.0%
freehold rental growth, g_1	13.0%
All risks yield of freehold income, k_1	13.38303%

Table 4: Value parameters at 2 year to the next rent review

Parameter	Value
Equivalent yield valuation	₦8,541,010.09
Explicit DCF valuation	₦8,541,010.09
Leasehold Equivalent yield, k_e	11.94054523%
All risks yield of leasehold income, k_0	9.30909159%
Leasehold rental growth, g_0	18.0%
freehold rental growth, g_1	13.0%
All risks yield of freehold income, k_1	13.38303%

Table 5: Value parameters at 1 year to the next rent review

Parameter	Value
Equivalent yield valuation	₦8,266,305.88
Explicit DCF valuation	₦8,266,305.88
Leasehold Equivalent yield, k_e	12.52225966%
All risks yield of leasehold income, k_0	9.30909159%
Leasehold rental growth, g_0	18.0%
freehold rental growth, g_1	13.0%
All risks yield of freehold income, k_1	13.38303%

Table 6: Value parameters at full reversion to leaseholder

Parameter	Value
Equivalent yield valuation	₦7,867,964.66
Explicit DCF valuation	₦7,867,964.66
Leasehold Equivalent yield, k_e	13.51423732%
All risks yield of leasehold income, k_0	9.30909159%
Leasehold rental growth, g_0	18.0%
freehold rental growth, g_1	13.0%
All risks yield of freehold income, k_1	13.38303%

For reversionary leaseholds, there is a negative relationship between number of years to the next rent revision and the leasehold equivalent yield. Similarly is the observation of a drastic decline in the capital value of leasehold interest as it approaches reversion.

4.5 *Discussion of Results*

The application of equivalent yield valuation technique to leasehold investment properties appears to be loathed by valuation scholars over the years on the condition that it belongs to the family of the conventional techniques that have come under serious criticism (Baum & Crosby, 2008; Baum *et al.*, 2011; Wood, 1986b). This may have informed its neglect in the construction of a synergized model aimed at determining implied rental growth rate of leaseholds valued part-way through rent review periods. However, insight into the existence of leasehold equivalent yield was provided in the seminal work of Udo (1989). Equivalent yield technique was recommended by Brown and Matysiak (2000) as the only conventional technique that can bridge the gap between growth implicit- and growth explicit DCF valuation of reversionary property investments. It is on the basis of these feats that this study attempted to set out an appropriate model for the determination of input parameters of explicit leasehold DCF appraisal. Just as in the case of reversionary freeholds exemplified by Brown and Matysiak (2000) and Wyatt (2013), the reversionary leasehold equivalent yield valuation model was equated with the reversionary leasehold growth explicit DCF valuation model, to form hybrid DCF model from where four unknown variables comprising the all risks yield and the implied growth rates of leasehold cash inflows and cash outflows were calculated. The tools used to perform the scenario analysis that lead to the determination of appropriate growth rates for leasehold cash inflows and cash outflows comprise Goal Seek- and Solver, which are specialized What-If analysis functions in Excel®.

The appropriate test of an optimal valuation in this case is the one that exhibits zero slack in the iterated and calculated implied growth rates for cash inflows and cash outflows respectively. From the hypothetical reversionary leasehold valuation case study, the sale of the leasehold interest for ₦8,704,728.55 might have been driven by a market implied growth rates of 18% and 13% for rent received and rent payable respectively. On the other hand, the purchase of the same leasehold interest for ₦8,704,728.55 might have been influenced by an expectation of market implied growth rates of 18% and 13% for cash inflows and cash outflows respectively. Although existing seminal works of Brown and Matysiak (2000) and Wyatt (2013) provided the framework and technique for the determination of a "single" implied rental growth rate of freehold investment properties valued part-way through a rent review period, this study is an extension of the same framework and technique to the determination of implied growth rates of leasehold cash inflows and cash outflows respectively.

5. Conclusion

This study examined the use of spreadsheet-assisted scenario analysis tools and techniques to determine rental growth rates of leasehold investment properties valued part-way through rent review periods. In consonance with the first objective of study, it was found that a combination of the leasehold equivalent yield valuation model and the leasehold DCF valuation model to form what is tagged "the hybrid leasehold DCF model" is required for the identification of input parameters for determining growth rates of cash inflow and cash outflow respectively. It was on this basis that a contemporary leasehold valuation problem where the market price, contract rent received and payable, market rent received payable, review period of cash inflow and cash outflow, number of years to the next revision of cash inflow and cash outflow, unexpired term of head lease, and equated yield are known was put forward. Thirdly, the tools and procedure for scenario analysis leading to the determination of appropriate growth rates for leasehold cash inflows and cash outflows were derived from the basic theories of linear optimization and the Solver scenario analysis tool in Excel®. In consonance with the fourth objective of study, the four unknown variables that were calculated within the leasehold DCF valuation scenario include all risks yields and growth rates for cash inflow and cash outflow respectively.

With recourse to the first research question and a valuation case study, growth rates of leasehold cash inflow and cash outflow were determined using Excel Solver which is a What-if scenario analysis tool capable of solving multiple output variables arising from changes in multiple input variables in a valuation. A total of eleven scenarios were generated to help identify the optimal solution for these parameters. In response to the second research question and the valuation case study, the leasehold capital value of ₦8,704,728.55 was likely driven by expectation of market implied growth rates of 18% and 13% for cash inflows and cash outflows respectively.

The use of spreadsheet-assisted scenario analysis with the hybrid DCF appraisal model to determine the rental growth rate of leasehold investment properties valued part-way through a rent review period presents some implications for leasehold investors. For instance, the market for leasehold investments responds to the gradual termination of the interest by compensating the holder of such interest with higher weighted average income yield in lieu of diminishing capital value. Therefore, investors seeking income returns and income growth for a limited time horizon might opt for leasehold investments provided an appraisal of viable cash flow growth rates can be established.

Just as in the case of freehold investments, the use of spreadsheet-assisted scenario analysis and techniques can possibly address the question of rental growth rates that justifies the discounted cash inflows and outflows required to produce a desired outcome for leasehold investment properties valued part-way through rent review periods.

This study was conducted within the framework of spreadsheet and DCF valuation of leaseholds. It would be recalled that Adams *et al.* (2003) underscored the deficiency of DCF valuation technique to handle the complexity of options in lease contracts. Therefore, further research should evolve appropriate techniques for the determination of implied rental growth rate of leasehold cash inflows and cash outflows characterized by embedded options and how these techniques can be deployed in the appraisal of leasehold investment property.

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